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MASS TRANSFER BETWEEN A SINGLE DROP AND A CONTINUOUS PHASE

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Abstract—The time dependent convective-diffusion equations for mass transfer between a drop and a continuous phase are solved in two cases: (1) the case of small Reynolds numbers and (2) the case of potential flow. The equations are solved by means of a similarity variable $\eta_i = y/\delta_i(\theta, t)$ which enable their transformation into an ordinary differential equation for the concentration $c_i = c_i(\eta_i)$ and a first order equation with partial derivatives for $\delta_i = \delta_i(\theta, t)$. Equations for the mass-transfer coefficient for the unsteady and steady states are obtained. The time in which the steady state is reached is evaluated.

NOMENCLATURE

a,	drop radius;
A_i, B_i,	constants given by equations (20–23);
c_i,	concentration;
$c_{i,0}$,	value of c_i for $t = 0$ and for $\|y\| \to \infty$;
D_i,	diffusion coefficient;
H,	equilibrium dissolution constant;
K_i,	mass-transfer coefficient;
N_θ,	mass flux;
N,	average mass flux over the drop surface;
Pe_i,	$= \frac{1}{4}(\mu'/\mu + \mu')\, Pe'_i$;
Pe'_i,	$= 2aU/D_i$, Péclet number;
Pe''_i,	$= \frac{3}{4}\, Pe'_i$;
r,	radial variable (spherical coordinate system);
Sh_i,	$= 2K_i a/D_i$, Sherwood number;
t,	time;
T,	$= \dfrac{1}{2}\dfrac{tU}{a}\dfrac{\mu'}{\mu + \mu'}$;
T',	$= \dfrac{3}{2}\dfrac{tU}{a}$;
U,	translational velocity of the drop;
$v_{r,i}$,	radial component of velocity with respect to the center of the drop;
v_θ,	tangential component of velocity with respect to the center of the drop;
v_0,	$= \dfrac{U}{2}\dfrac{\mu'}{\mu + \mu'}$;
y,	distance at the interface considered positive if directed toward the center of the drop;
Y,	$= y/a$.

Greek symbols

α_i, β,	constants;
δ_i,	length introduced by means of the similarity variable η_i;
Δ_i,	$= \delta_i/a$;
ε_i,	$= \Delta_i^2$;
η_i,	$= y/\delta_i$;
ϕ_i,	arbitrary functions;
ψ,	function defined by equation (35);
μ,	viscosity of the dispersed phase;
μ',	viscosity of the continuous phase;
τ_i,	$= tD_i/a^2$;
θ,	polar angle (spherical coordinate system).

Subscripts

1,	for the dispersed phase;
2,	for the continuous phase.

INTRODUCTION

THE PROBLEM of heat or mass transfer between a drop or a bubble and a continuous phase has

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been usually examined by assuming that the steady state is achieved and that the interface concentration of the drop is constant. Of the papers in which this problem is dealt with we mention only a few [1–5]. In two recent papers [6, 7] the time dependent convective diffusion equation has been solved numerically. Bentwich, Szwarcbaum and Sideman [6] have studied the case of mass transfer in the continuous phase assuming potential flow and Johns and Beckman [7] the case of mass transfer in the dispersed phase for small Reynolds numbers.

In the present paper the problem of mass transfer will be examined by solving analytically in a limiting case, the time dependent convective-diffusion equations of the dispersed and of the continuous phase. Two cases will be dealt with: the case of small Reynolds numbers and the case of potential flow. The problem will be solved by means of a similarity transformation suggested by the author in a previous paper [8].

MASS OR HEAT TRANSFER BETWEEN A SINGLE SPHERICAL DROP AND THE CONTINUOUS PHASE FOR $Re < 1$

The convective-diffusion equation has the form

$$\frac{\partial c_i}{\partial t} + v_{r,i}\frac{\partial c_i}{\partial r} + \frac{v_{\theta,i}}{r}\frac{\partial c_i}{\partial \theta} = D_i\left[\frac{\partial^2 c_i}{\partial r^2} + \frac{2}{r}\frac{\partial c_i}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial c_i}{\partial \theta}\right)\right] \quad (1)$$

where $i = 1$ refers to the dispersed phase and $i = 2$ to the continuous phase. For the velocity components Hadamard's equations are valid [3, 4]:

$$v_{r,1} = -v_0\left(1 - \frac{r^2}{a^2}\right)\cos\theta \quad (2)$$

$$v_{\theta,1} = v_0\left(1 - \frac{2r^2}{a^2}\right)\sin\theta \quad (3)$$

$$v_{r,2} = \left[\left(\frac{U}{2} - v_0\right)\left(\frac{r}{a}\right)^{-3} + \left(v_0 - \frac{3U}{2}\right)\left(\frac{r}{a}\right)^{-1} + U\right]\cos\theta \quad (4)$$

$$v_{\theta,2} = \left[\left(\frac{U}{4} - \frac{v_0}{2}\right)\left(\frac{r}{a}\right)^{-3} + \left(\frac{3U}{4} - \frac{v_0}{2}\right)\left(\frac{r}{a}\right)^{-1} - U\right]\sin\theta \quad (5)$$

where

$$v_0 = \frac{U}{2}\frac{\mu'}{\mu + \mu'}.$$

The diffusion coefficient in a liquid is very small. For this reason if the time of contact of an element of fluid with the interface is not too large, the depth of penetration by diffusion is also small. An element of liquid from the continuous phase remains in contact with the drop a very short time, of the order of $2a/U$, and continuously fresh elements of liquid are brought into contact with the interface. For this reason the depth of penetration by diffusion in these elements is very small.

In the dispersed phase there exists the circulation motion known as the "Hadamard's rings". Elements of liquid are brought into contact with the interface of the drop at the front stagnation point and are moving along the interface up to the rear stagnation point; in this point the motion is dirrected along the vertical up to the front stagnation point. Consequently the elements of liquid brought into contact with the interface are continually the same, fresh elements coming into contact with the interface only during the initial lapse of time equal to that needed for an element of fluid to pass through the distance between the rear and the front stagnation point. However, as long as the elements of fluid are in motion along the interface a process of mass transfer takes place between them and the liquid from the continuous phase, while as long as they are in motion along the vertical they exchange mass with the bulk of the dispersed phase. In this manner, though the elements of liquid brought into contact with the interface are the same, they are refreshed in some measure and the process of mass transfer at the interface occurs for not too large times approximately as if they would be fresh elements. There exists therefore for not too large values of the time an approximate similarity with the mass transfer in the dispersed phase.

Since the depth of penetration by diffusion is small the concentration varies appreciably in the immediate vicinity of the interface and is practically constant at somewhat larger distances. However, in the immediate vicinity of the interface $y \ll a$ and we may approximate the velocity distribution by means of the expressions obtained by expanding equations (2–5) in series with respect to y/a and by retaining only the first term.

In this manner one obtains

$$v_{r,i} = -2v_0 \frac{y}{a} \cos\theta, \tag{6}$$

$$v_{\theta,i} = -v_0 \sin\theta. \tag{7}$$

The calculations which follow are therefore valid if the depth of penetration is small as compared to the drop radius. Neglecting also, as usual, the terms

$$\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial c_i}{\partial\theta}\right)$$

and $(2/r)(\partial c_i/\partial r)$ as compared to $D_i\, \partial^2 c_i/\partial y^2$*, the following convective-diffusion equations results

$$\frac{\partial c_i}{\partial \tau_i} = \frac{\partial^2 c_i}{\partial Y^2} + Pe_i\left(-2Y\cos\theta \frac{\partial c_i}{\partial Y} + \sin\theta \frac{\partial c_i}{\partial\theta}\right), \quad i = 1, 2 \tag{8}$$

* We note that in the case examined here the average value of

$$\frac{D_i}{r^2 \sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial c_i}{\partial\theta}\right)$$

over the drop surface may be neglected compared to the average of $D_i\, \partial^2 c_i/\partial y^2$ and also that the average value of $(2D_i/a)(\partial c_i/\partial y)$ over the thickness δ_i of the diffusion boundary layer may be neglected compared to the average of $D_i\, \partial^2 c_i/\partial y^2$. To these conclusions one may arrive by using equation (18) for c_i.

where

$$\tau_i \equiv \frac{tD_i}{a^2} \tag{9a}$$

$$Pe_i \equiv \frac{1}{4} \frac{\mu'}{\mu + \mu'} \frac{2aU}{D_i} \tag{9b}$$

$$Y \equiv \frac{y}{a}. \tag{9c}$$

The following initial and boundary conditions must be satisfied:

$$\text{for } \tau_i = 0, \quad c_1 = c_{1,0} \text{ and } c_2 = c_{2,0} \tag{10}$$

$$\text{for } Y = 0, \quad c_1 = Hc_2 \text{ and } D_1 \frac{\partial c_1}{\partial Y} = D_2 \frac{\partial c_2}{\partial Y} \tag{11}$$

$$\text{for } Y \to \infty \quad c_1 \to c_{1,0} \tag{12a}$$

$$\text{for } Y \to -\infty \; c_2 \to c_{2,0}. \tag{12b}$$

The boundary conditions (12) are a result of the fact that the depth of penetration by diffusion is small and consequently the distribution of concentration in the vicinity of the interface of the drop may be approximated by that valid for a semi-infinite fluid. We not also the fact that the boundary condition (12a) implies the assumption that the elements of liquid brought into contact with the interface at the front stagnation point have the initial concentration $c_{1,0}$ (see also the above discussion).

Equations (8) and the initial and boundary conditions (10–12) are compatible with solutions of the form

$$c_i = c_i(\eta_i) \tag{13}$$

where

$$\eta_i = \frac{y}{\delta_i} = \frac{Y}{\Delta_i} \qquad \Delta_i = \Delta_i(\theta, \tau_i). \tag{14}$$

The new variable η_i enables the transformation of equations (8) into

$$\frac{d^2 c_i}{d\eta_i^2} + \eta_i \frac{dc_i}{d\eta_i}\left[\frac{1}{2}\frac{\partial \Delta_i^2}{\partial \tau_i} - 2(Pe_i \cos\theta)\,\Delta_i^2 - \frac{Pe_i}{2}\sin\theta \frac{\partial \Delta_i^2}{\partial\theta}\right] = 0. \tag{15}$$

In order that $c = c_i(\eta_i)$ and $\Delta_i = \Delta_i(\tau_i, \theta)$, we must have

$$\frac{1}{2}\frac{\partial \Delta_i^2}{\partial \tau_i} - 2(Pe_i \cos\theta)\,\Delta_i^2 - \frac{Pe_i}{2}\sin\theta \frac{\partial \Delta_i^2}{\partial \theta} = \beta \quad (16)$$

$$\frac{d^2 c_i}{d\eta_i^2} + \beta\eta_i \frac{dc_i}{d\eta_i} = 0, \quad (17)$$

β being an arbitrary positive constant. Selecting $\beta = 2$, the solutions of equations (17) are

$$c_1 = A_1 \int_0^{\eta_1} e^{-x^2}\,dx + B_1 \quad (18)$$

$$c_2 = A_2 \int_0^{\eta_2} e^{-x^2}\,dx + B_2. \quad (19)$$

The boundary conditions (11) and (12) enable the determination of the four constants. One obtains

$$A_1 = \frac{2}{\sqrt{\pi}} \frac{\left(\frac{c_{1,0}}{H} - c_{2,0}\right)}{\left(\frac{D_1}{D_2}\right)^{\frac{1}{2}} + \frac{1}{H}} \quad (20)$$

$$B_1 = \frac{c_{1,0} + c_{2,0}\left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}}{1 + \frac{1}{H}\left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}} \quad (21)$$

$$A_2 = \frac{2}{\sqrt{\pi}} \frac{\frac{c_{1,0}}{H} - c_{2,0}}{1 + \frac{1}{H}\left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}} \quad (22)$$

$$B_2 = \frac{c_{1,0} + c_{2,0}\left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}}{H + \left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}} \quad (23)$$

In the establishment of the expressions of the constants A_i and B_i it is an important fact, to be shown below [see equation (30)] that $\delta_1/\delta_2 = (D_1/D_2)^{\frac{1}{2}}$.

The functions Δ_i are determined by solving equations

$$\frac{\partial \varepsilon_i}{\partial \tau_i} - 4(Pe_i \cos\theta)\,\varepsilon_i - Pe_i \sin\theta \frac{\partial \varepsilon_i}{\partial \theta} = 4, \quad (24)$$

where

$$\varepsilon_i \equiv \Delta_i^2 \quad (25)$$

for the boundary condition

$$\varepsilon_i = 0 \quad \text{for} \quad \tau_i = 0. \quad (26)$$

The last boundary conditions result as a consequence of the initial condition (10). Indeed, only if $\Delta_i = 0$ for $\tau_i = 0$, equations (18) and (19) satisfy the initial conditions (10).

The characteristic system which may be attached to equation (24) has the form

$$\frac{d\tau_i}{1} = -\frac{d\theta}{Pe_i \sin\theta} = \frac{d\varepsilon_i}{4(Pe_i \cos\theta)\,\varepsilon_i + 4}. \quad (27)$$

From the first equation one obtains

$$Pe_i\tau_i + \ln\tan\frac{\theta}{2} = \alpha_i$$

and from the second

$$\varepsilon_i = \frac{1}{\sin^4\theta}\left[\alpha_i' + \frac{4}{Pe_i}(\cos\theta - \tfrac{1}{3}\cos^3\theta)\right]$$

Therefore

$$\varepsilon_i \sin^4\theta - (4/Pe_i)(\cos\theta - \tfrac{1}{3}\cos^3\theta) = \phi_i(Pe_i\tau_i + \ln\tan\theta/2), \quad (28)$$

where ϕ_i is an arbitrary function.

The form of the functions ϕ_i is determined by taking into account the boundary conditions (26). For $\tau_i = 0$ we have

$$\left(\phi_i \ \ln\tan\frac{\theta}{2}\right) = -\frac{4}{Pe_i}(\cos\theta - \tfrac{1}{3}\cos^3\theta).$$

However

$$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \equiv \frac{1-\exp\left(2\ln\tan\frac{\theta}{2}\right)}{1+\exp\left(2\ln\tan\frac{\theta}{2}\right)}$$

and therefore

$$\phi_i\left(Pe_i\tau_i + \ln\tan\frac{\theta}{2}\right) = -\frac{4}{Pe_i}\left[\frac{1-\exp\left[2\left(\tau_i Pe_i + \ln\tan\frac{\theta}{2}\right)\right]}{1+\exp\left[2\left(\tau_i Pe_i + \ln\tan\frac{\theta}{2}\right)\right]} - \frac{1}{3}\left(\frac{1-\exp\left[2\left(\tau_i Pe_i + \ln\tan\frac{\theta}{2}\right)\right]}{1+\exp\left[2\left(\tau_i Pe_i + \ln\tan\frac{\theta}{2}\right)\right]}\right)^3\right]$$

Consequently

$$\Delta_i^2\sin^4\theta = \frac{4}{Pe_i}\left\{\cos\theta - \tfrac{1}{3}\cos^3\theta - \left[\frac{1-\tan^2\frac{\theta}{2}\exp(2\tau_i Pe_i)}{1+\tan^2\frac{\theta}{2}\exp(2\tau_i Pe_i)} - \frac{1}{3}\left(\frac{1-\tan^2\frac{\theta}{2}\exp(2\tau_i Pe_i)}{1+\tan^2\frac{\theta}{2}\exp(2\tau_i Pe_i)}\right)^3\right]\right\} \tag{29}$$

We remark the fact that the product

$$\tau_i Pe_i = \frac{D_i t}{a^2}\frac{1}{4}\frac{\mu'}{\mu+\mu'}\frac{2aU}{D_i} = \frac{1}{2}\frac{tU}{a}\frac{\mu'}{\mu+\mu'} \equiv T$$

does not depend on the index i. For this reason equation (29) leads for the ratio δ_1/δ_2 to the equation

$$\frac{\delta_1}{\delta_2} = \left(\frac{D_1}{D_2}\right)^{\frac{1}{2}} \tag{30}$$

For the mass flux N_θ one obtains

$$N_\theta = -D_1\left(\frac{\partial c_1}{\partial y}\right)_{y=0} = \frac{2}{\sqrt{\pi}}\frac{c_{2,0}-\frac{c_{1,0}}{H}}{\left(\frac{D_1}{D_2}\right)^{\frac{1}{2}}+\frac{1}{H}}\frac{D_1}{a}\left(\frac{Pe_1}{4}\right)^{\frac{1}{2}} \times \frac{\sin^2\theta}{\left[\cos\theta - \frac{1}{3}\cos^3\theta - \frac{1-\tan^2\frac{\theta}{2}e^{2T}}{1+\tan^2\frac{\theta}{2}e^{2T}} + \frac{1}{3}\left(\frac{1-\tan^2\frac{\theta}{2}e^{2T}}{1+\tan^2\frac{\theta}{2}e^{2T}}\right)^3\right]^{\frac{1}{2}}} \tag{31}$$

The average mass flux over the surface of the drop is given by

$$N = \frac{1}{4\pi a^2}\int_0^\pi 2\pi a^2 N_\theta \sin\theta\,d\theta$$

$$= \frac{D_1}{a}\left(\frac{Pe_1}{4\pi}\right)^{\frac{1}{2}}\frac{Hc_{2,0}-c_{1,0}}{H\left(\frac{D_1}{D_2}\right)^{\frac{1}{2}}+1}\int_0^\pi \frac{\sin^3\theta\,d\theta}{\left[\cos\theta - \frac{1}{3}\cos^3\theta - \frac{1-\tan^2\frac{\theta}{2}e^{2T}}{1+\tan^2\frac{\theta}{2}e^{2T}} + \frac{1}{3}\left(\frac{1-\tan^2\frac{\theta}{2}e^{2T}}{1+\tan^2\frac{\theta}{2}e^{2T}}\right)^3\right]^{\frac{1}{2}}}. \tag{32}$$

If one introduces an overall mass-transfer coefficient K_1, defined by

$$K_1 = \frac{N}{Hc_{2,0} - c_{1,0}}, \tag{33}$$

equation (32) may be written under the form

$$Sh_1 \equiv \frac{2K_1 a}{D_1} = \left(\frac{Pe_1}{\pi}\right)^{\frac{1}{2}} \frac{1}{H\left(\frac{D_1}{D_2}\right)^{\frac{1}{2}} + 1} \psi(T) \tag{34}$$

where

$$\psi(T) \equiv \int_0^\pi \frac{\sin^3\theta \, d\theta}{\left[\cos\theta - \frac{1}{3}\cos^3\theta - \dfrac{1 - \tan^2\frac{\theta}{2} e^{2T}}{1 + \tan^2\frac{\theta}{2} e^{2T}} + \dfrac{1}{3}\left(\dfrac{1 - \tan^2\frac{\theta}{2} e^{2T}}{1 + \tan^2\frac{\theta}{2} e^{2T}}\right)^3\right]^{\frac{1}{2}}}. \tag{35}$$

Table 1

T	ψ
0·01	19·930
0·10	6·340
0·30	3·740
0·50	3·014
1	2·461
2	2·315
3	2·309
5	2·309
10	2·309
100	2·309

For an overall mass-transfer coefficient K_2, defined by

$$K_2 = \frac{N}{c_{2,0} - \dfrac{c_{1,0}}{H}}$$

one obtains

$$Sh_2 \equiv \frac{2K_2 a}{D_2} = \left(\frac{Pe_2}{\pi}\right)^{\frac{1}{2}} \frac{1}{\frac{1}{H}\left(\frac{D_2}{D_1}\right)^{\frac{1}{2}} + 1} \psi(T). \tag{36}$$

Several limiting cases may be evidenced:

(1) If $D_2 \gg D_1$ the diffusion in the dispersed phase is the rate determining step and equation (34) leads to

$$Sh_1 = \left(\frac{Pe_1}{\pi}\right)^{\frac{1}{2}} \psi(T). \tag{37}$$

If $\tau_i \to 0$, equation (37) leads to

$$Sh_1 = \frac{2}{\sqrt{\pi}} \frac{1}{\tau_i^{\frac{1}{2}}}, \tag{37'}$$

i.e. to the result of Higbie's penetration theory.

For $T \gg 1$, equation (37) leads to

$$Sh_1 = \frac{4}{\sqrt{3}} \left(\frac{Pe_1}{\pi}\right)^{\frac{1}{2}}. \tag{37''}$$

Equation (37″) is valid for a hypothetical steady state which would be achieved if the amount of mass transferred along the interface were equal to that exchanged by the elements of fluid with the bulk of the dispersed phase (so that the elements of liquid brought into contact with the front stagnation point had continuously the concentration $c_{1,0}$).

(2) If $D_1 \gg D_2$ the diffusion in the continuous phase is the rate determining step and equation (36) leads to

$$Sh_2 = \left(\frac{Pe_2}{\pi}\right)^{\frac{1}{2}} \psi(T).$$

If $D_1 \gg D_2$ and $T \gg 1$

$$Sh_2 = \frac{4}{\sqrt{(3\pi)}}(Pe_2)^{\frac{1}{2}} \tag{38}$$

The steady state for which equation (38) is valid is achieved after a time given by $T \approx 5$.

It is of interest to note that the interface concentration is constant and that the overall mass-transfer coefficient may be obtained by the usual law of additivity of the resistances.

It may also be stressed that in the framework of the approximations used here the equations for the mass-transfer coefficient are the same for the continuous and for the dispersed phase. A qualitative explanation of this result has been given above.

The main approximation used above is $\delta_i \ll a$. It is not difficult to show that such an inequality is valid if $Pe_i \gg 1$.

MASS OR HEAT TRANSFER BETWEEN A SINGLE SPHERICAL DROP AND A CONTINUOUS PHASE FOR THE POTENTIAL FLOW

In the vicinity of a fluid boundary and for sufficiently large Reynolds number the role of the viscosity is not too important and for this reason it is possible to approximate the velocity distribution by that valid for potential flow.

In this case we have

$$v_{r,i} = U\left(1 - \frac{a^3}{r^3}\right)\cos\theta \tag{39}$$

$$v_{\theta,i} = -U\left(1 + \frac{a^3}{2r^3}\right)\sin\theta. \tag{40}$$

Since in the region of interest $y \ll a$, we may approximate the above equations by

$$v_{r,i} = -3U\frac{y}{a}\cos\theta, \tag{41}$$

$$v_{\theta,i} = -\tfrac{3}{2}U\sin\theta. \tag{42}$$

Equation (1) becomes in this case (after the above mentioned simplifying assumptions are made)

$$\frac{\partial c_i}{\partial \tau_i} = \frac{\partial^2 c_i}{\partial Y^2} + Pe_i''\left(-2Y\cos\theta\frac{\partial c_i}{\partial Y} + \sin\theta\frac{\partial c_i}{\partial\theta}\right) \tag{43}$$

where

$$Pe_i'' \equiv \frac{3}{2}\frac{aU}{D_i}. \tag{44}$$

Equations (43) being of the same form as equations (8) all the above equations obtained for the mass-transfer coefficient may be extended to this case too, if Pe_i is replaced by Pe_i''.

In this manner we get

$$Sh_1 = \left(\frac{Pe_1''}{\pi}\right)^{\frac{1}{2}}\frac{1}{H\left(\frac{D_1}{D_2}\right)^{\frac{1}{2}} + 1}\psi(T') \tag{45}$$

and

$$Sh_2 = \left(\frac{Pe_2''}{\pi}\right)^{\frac{1}{2}}\frac{1}{\frac{1}{H}\left(\frac{D_2}{D_1}\right)^{\frac{1}{2}} + 1}\psi(T') \tag{46}$$

where

$$T' \equiv Pe_i''\tau_i \equiv \frac{3}{2}\frac{aU}{D_i}\frac{tD_i}{a^2} = \frac{3}{2}\frac{tU}{a}. \tag{47}$$

It may be verified that in the limiting case of $D_1 \gg D_2$ and $T' \gg 1$ (steady state) one obtains, as expected, Boussinesq's equation

$$Sh_2 = \frac{2}{\sqrt{\pi}}\left(\frac{2aU}{D_2}\right)^{\frac{1}{2}}. \tag{48}$$

The steady state is achieved after a time $T' \approx 5$.

An equation of the same form is obtained in the limiting case $D_2 \gg D_1$ and $T' \gg 1$

$$Sh_1 = \frac{2}{\sqrt{\pi}}\left(\frac{2aU}{D_1}\right)^{\frac{1}{2}}. \tag{49}$$

Equation (49) is valid for a hypothetical steady state similar to that discussed above in connection with small Reynolds numbers.

CONCLUSION

Exact analytical solutions have been obtained for the time-dependent convective-diffusion equations in the case of mass transfer from spherical drops for the cases in which the depth of penetration by diffusion is very small. The method is based on a similarity variable $\eta_i = y/\delta_i(t, \theta)$ which enables the transformation of the second order equation with partial derivatives into an ordinary equation for the concentration and a first order equation with partial derivatives for $\delta_i = \delta_i(t, \theta)$.

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Résumé—Les équations de la diffusion avec convection dépendant du temps pour le transport de masse entre une goutte et une phase continue sont résolues dans deux cas: (1) le cas des petits nombres de Reynolds, et (2) le cas de l'écoulement potentiel. La résolution est effectuée au moyen d'une variable de similitude $\eta_i = y/\delta_i(\theta, t)$ qui permet de les transformer en une équation différentielle ordinaire pour la concentration $c_i = c_i(\eta_i)$ et une équation aux dérivées partielles du premier ordre pour $\delta_i = \delta_i(\theta, t)$. Les équations donnant le coefficient de transport de masse pour les états stationnaire et non-stationnaire sont obtenues. Le temps au bout duquel l'état permenent ets atteint est évalué.

Zusammenfassung—Die zeitabhängigen Konvektions-Diffusions-Gleichungen für den Stofftransport zwischen einem Tropfen und einer kontinuierlichen Phase sind für zwei Fälle gelöst: (1) für den Fall kleiner Reynolds-Zahlen und (2) für den Fall der Potentialgleichung. Die Gleichungen wurden mit Hilfe einer Ähnlichkeitsvariablen $\eta_i = y/\delta_i(\theta, t)$ gelöst, welche ihre Transformation in gewöhnliche Differentialgleichungen für die Konzentration $c_i = c_i(\eta_i)$ und in eine Gleichung erster Ordnung mit partiellen Ableitungen für $\delta_i = \delta_i(\theta, t)$ ermöglicht. Gleichungen für den Stoffübergangskoeffizienten für stationäre und instationäre Zuständ liessen sich erhalten. Die Zeit, nach welcher der stationäre Zustand erreicht ist, wird berechnet.

Аннотация—Решены уравнения конвективной диффузии, зависящей от времени, для переноса массы между каплей и сполшной фазой в двух случаях: (1) при небольших числах Рейнольдса и (2) для случая потенциального потока. Уравнения решаются с помощью подстановки $\eta_i = y/\delta_i(\theta, t)$, что позволяет преобразовать данные уравнения в обычные дифференциальные уравнения для концентрации $c_i = c_i(\eta_i)$ и в уравнение первого порядка в частных производных для $\delta_i = \delta_i(\theta, t)$. Получены уравнения для коэффициента массообмена в стационарных и нестационарных условиях. Определено время установления стационарного состояния.